CONSENSUS FORMATION AND CHANGE ARE ENHANCED BY NEUTRALITY

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SUMMARY

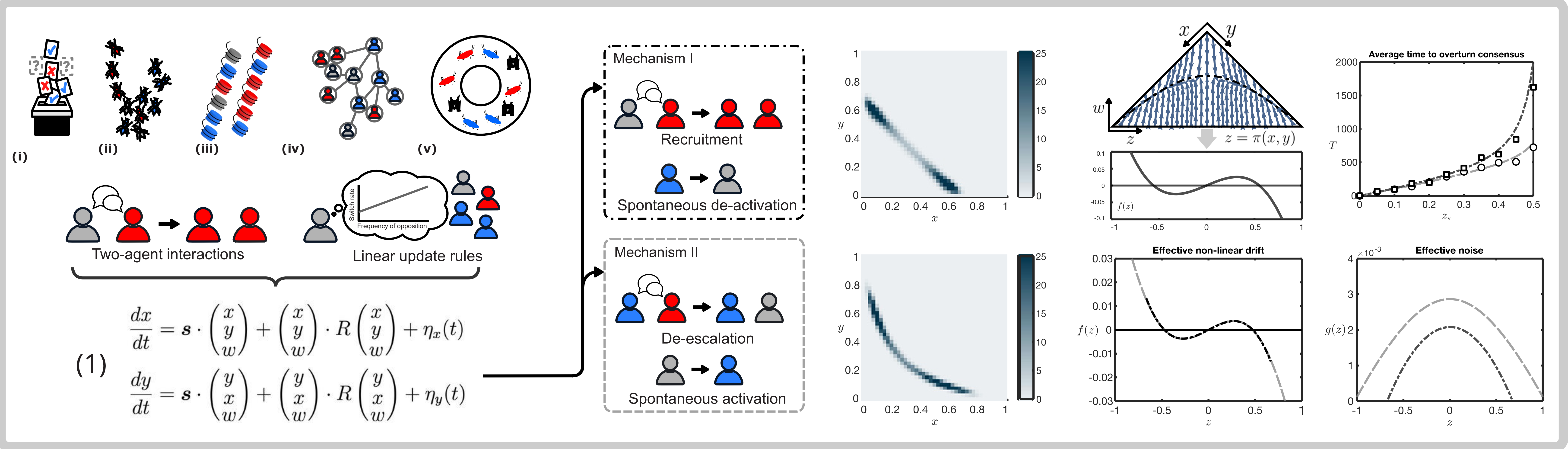
A minimal three-state, symmetric model with only linear pairwise responses explains how neutrality both enables consensus formation and facilitates consensus switching via two universal slow-manifold regimes, as validated by locust collective motion data and a human voting game with abstention.

MOTIVATION

Effective collective decision-making in human and animal groups requires mechanisms for consensus formation and change, typically via feedback loops in which individuals adapt their behaviour and opinions based on their perception of others. However, levels of participation can be highly variable over time, with individuals sometimes adopting neutral positions.

MODEL OVERVIEW

Agents occupy symmetric active states X, Y, or neutral W, with total population N fixed. We assume a linear response model for the transition rates.



$$(1)\quad \frac{dx}{dt} = \boldsymbol{s}\cdot\begin{pmatrix}x\\y\\w\end{pmatrix} + \begin{pmatrix}x\\y\\w\end{pmatrix}\cdot R\begin{pmatrix}x\\y\\w\end{pmatrix} + \eta_x(t)$$

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Fig. 1 Our model describes a broad class of symmetric binary decision models with neutral agents that can be used to represent a wide range of processes e.g. (i) voting with abstention (ii) insect foraging (iii) nucleosome conformal modification (iv) social network dynamics (v) onset of collective motion. (Left) We consider at most two-body interactions or, equivalently, linear rates of switching. General dynamics on the 2-dimensional simplex with rates s for spontaneous state changes and R for reactive changes. The dominant reaction dynamics for the two main mechanisms of consensus formation - based on either (i) recruitment (for Mechanism I) or (ii) de-escalation (for Mechanism II) interactions. (Middle-right) Dynamics associated with each mechanism. (Top) Linear manifold (black dashed line). (Bottom) Hyperbolic manifold (grey dashed line). In each case, the heat map shows the simulated residency duration at each point in state space; the stochastic dynamics remain close to the predicted slow manifolds. (Right) A timescale separation allows us to project from the simplex to effective non-linear dynamics of z. The amplified effect of noise in Mechanism II allows for faster overturning of consensus. In the comparisons, parameters are chosen so that the effective drifts align with Mechanism I in dark grey, and Mechanism II in light grey. Effective noise is much larger for Mechanism II than Mechanism I leading to time to overturn consensus as a function of polarisation at consensus being higher for Mechanism I than Mechanism II.

CASE STUDY: LOCUSTS

Reanalysis of ring-arena experiments shows collective direction switches coincide with spikes in stopped (neutral) individuals, following curved, high-neutrality paths consistent with Mechanism II. Linear rate functions fit individual transitions, and the effective model's stationary distribution and manifold curvature estimated from data agree with the theory for groups of different sizes.

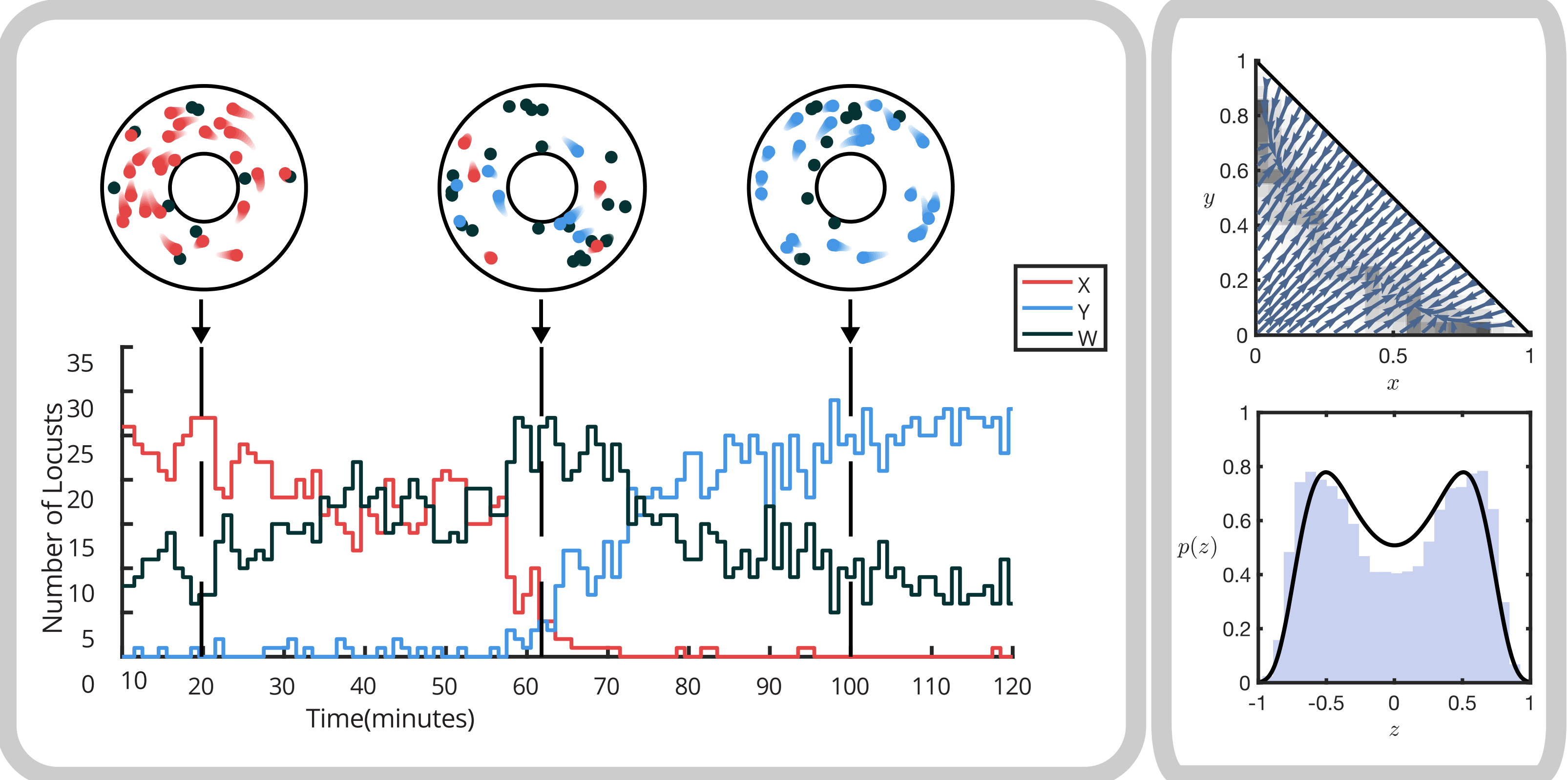

Fig. 2 Locust marching dynamics. (Left) An experimental time-course and snapshots (N = 35 locusts) showing a consensus switch between predominantly clockwise (red) and anti-clockwise (blue) motion, with an increase in neutral (stopped - black) individuals during the transient. (Right) Model fitted to data (see text) for experiments with N = 25 locusts. (Top) Two-dimensional state space residency distribution (greyscale heat map) with overlaid model flow fields (blue arrows). (Bottom) The corresponding one-dimensional projection of the stationary distribution from the experiment (blue histogram) onto the slow manifold and the model-predicted stationary probability distribution (black curve) for experiments.

WHY NEUTRALITY HELPS

Neutrality is both the cause and consequence of switching: perceived uncertainty and costs induce abstention, which depolarises the active pool, further increasing uncertainty and neutrality in a feedback loop that speeds switching via larger effective noise.

Conversely, neutrality also facilitates consensus formation by reducing direct conflict between polarised options, allowing interior consensus without higher-order microscopic interactions.

TAKE-HOME MESSAGES

- Only two slow-manifold pathways exist in symmetric three-state systems: constant neutrality or increased neutrality during switches. For both the locust and voting data, increased neutrality has been observed as the main mechanism for consensus switching.
- Neutrality is a lever for both stabilising consensus and accelerating overturn, with practical relevance from animal groups to voting and epigenetics.

ETHICS AND ACKNOWLEDGEMENTS

Human studies followed institutional approvals with controlled online deployment, attention checks, compensation, and anonymised data used for inference and model validation.

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CASE STUDY: VOTING GAME

In an iterated voting game with an abstain option, groups switch consensus along the hyperbolic slow manifold. Groups of about nine participants repeatedly choose X, Y, or Abstain with 10-second decisions, where voting costs 100 points and voting with the majority pays 150. Each participant sees a personalised sample of nine votes from the previous round, replaced uniformly with probability 0.4 to avoid fixation and elicit switches.

Data-fitting strongly favours the curved manifold as determined by residuals and Bayesian Information Criterion (BIC) for model selection, with significant z-tests for non-zero curvature. Removing abstention reduces switching or yields fragile consensus, confirming neutrality's role in both forming and overturning consensus efficiently.

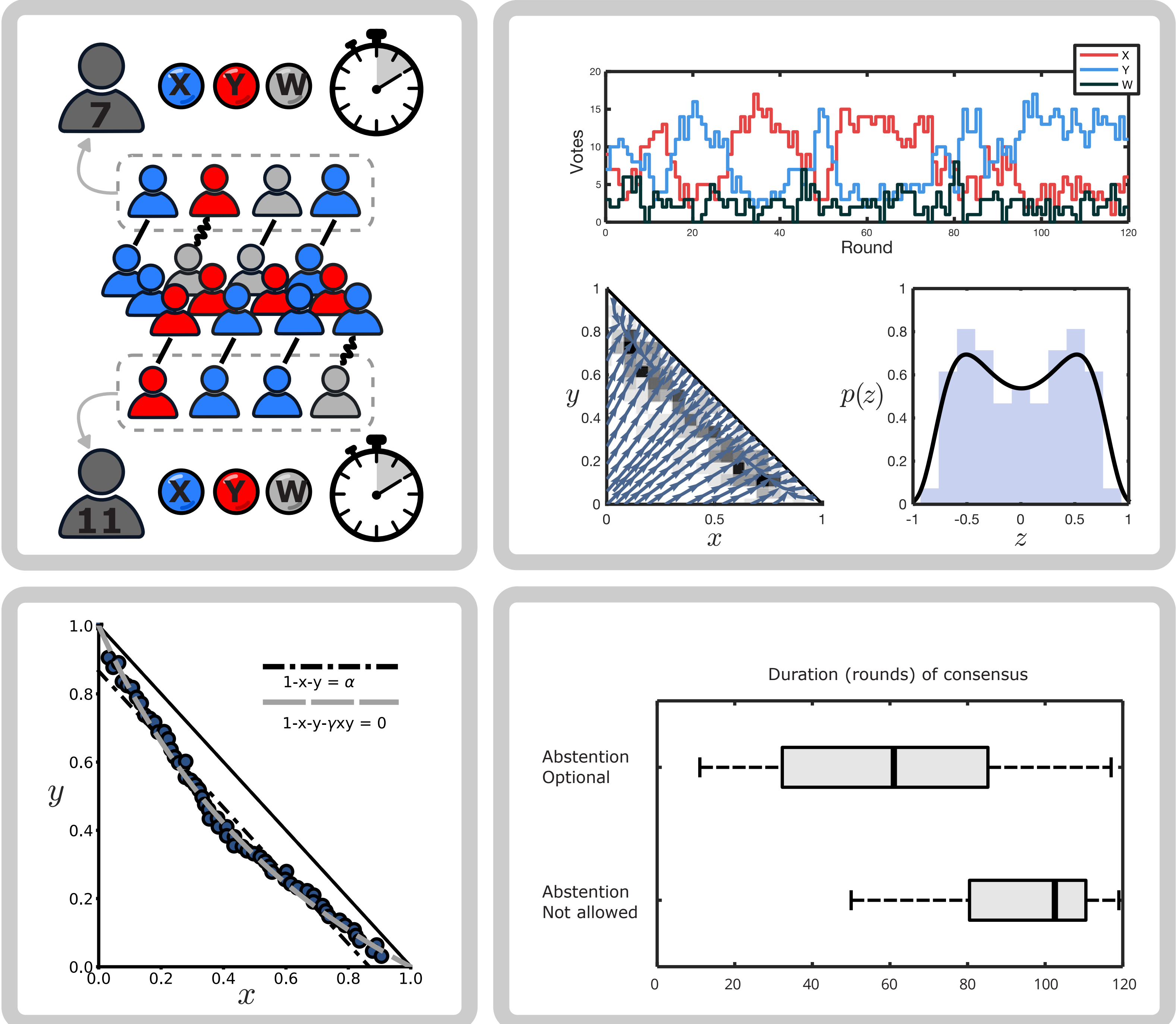

Fig. 3 Human voting dynamics. (Top-left) Two examples of samples of votes (avatars inside the dotted line) of participants from the previous round (indicated by the 12 coloured avatars in the middle), shown to individuals (dark grey) in an example of our human voting experiments with 12 participants. Most of the uniformly sampled participants' votes are displayed faithfully (straight black lines) while some are stochastically flipped (wiggly black lines). Individuals are then allowed to vote for option X, option Y, or to abstain (by choosing option W or not making a choice) within each round's ten-second voting period. (Top-right) An experiment with N = 19 participants. Time courses of the experimental data. Votes for option X, option Y and abstentions are represented in blue, red and black, respectively. A heat map of the data from the experimental time course with the flow field of the corresponding deterministic model trajectories (with parameters inferred from the experimental data) superimposed. A one-dimensional projection of the stationary probability distributions over the single variable z = x-y (blue bars). The theoretical stationary probability distribution from the model with experimentally inferred parameters is superimposed as a black curve. (Bottom-left) The curved manifold is a superior fit to the data than the straight manifold. (Bottom-right) Duration of periods of observed consensus in experiments with abstaining option (upper) and with no abstention option given (lower). For each replicate in both conditions, the duration of consensus is measured as the sum of the periods between passing from z > 0.75 to z < −0.75 (or vice versa) or until the end of the experiment at round 120.

RELEVANT LITERATURE

Sontag, A., Hoffmann, J., Rogers, T., Yates, C. A., "Consensus formation and change are enhanced by neutrality". (Under Review)
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Buhl, C., Sumpter, D. J., Couzin, I. D., Hale, J. J., Despland, E., Miller, E. R., and Simpson, S. J., Science 312, 1402 (2006).